Appendix: A Simple Staffing Method for Multi-Skill Call Centers

Auke Pot, Sandjai Bhulai & Ger Koole
Vrije Universiteit, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands

Supplementary Material

1 Additional Numerical Examples

This manuscript provides additional examples of the method from Pot, Bhulai, & Koole [PBK06].

1.1 Call Center with Three Skills

We consider an example with 3 skills and 7 skill sets. The skills of the agent groups are: $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3\}$, $S_4 = \{1, 2\}$, $S_5 = \{2, 3\}$, $S_6 = \{1, 3\}$, and $S_7 = \{1, 2, 3\}$. The routing is $(1 : 1 \to 4 \to 6 \to 7)$, $(2 : 2 \to 4 \to 5 \to 7)$, and $(3 : 3 \to 5 \to 6 \to 7)$. Jobs arrive with rates $\lambda_1 = \lambda_2 = \lambda_3 = 2$ and the service rates μ_{mg}, denoting the service rate of agents from group g working on call type m, are: $\mu_{11} = 0.3$, $\mu_{22} = 0.4$, $\mu_{33} = 0.5$, $\mu_{14} = 0.27$, $\mu_{24} = 0.36$, $\mu_{25} = 0.36$, $\mu_{35} = 0.45$, $\mu_{16} = 0.27$, $\mu_{36} = 0.45$, $\mu_{17} = 0.24$, $\mu_{27} = 0.32$, and $\mu_{37} = 0.40$. The costs of the agents are: $c_1 = 1.3$, $c_2 = 1.4$, $c_3 = 1.5$, $c_4 = 1.4$, $c_5 = 1.6$, $c_6 = 1.5$, and $c_7 = 1.6$.

A summary of the execution of the algorithm is depicted in Table 1. The meaning of the content needs no further explanation because the structure of the table is similar to Table 1 in [PBK06].

The table shows that no feasible solution is found when taking $n = 21$. Even for high values of $\tilde{\beta}$ the service level does not exceed 78%, while 80% is required. The best solution is obtained by setting $n = 22$, which resulted in a service level of exactly 80% and staffing costs of 30.6. Higher values of n yielded higher costs.

1.2 Call Center with Five Skills

The examples in this section concern a call center with 5 job types. Its agent groups and agent selection policy are similar to the ones used in Cezik & L'Ecuyer [CL06], which is depicted in Table 2. In contrast, an agent serves the longest waiting customer at a service completion.

n											
21			22			23			24		
$\tilde{\beta}$	s^*	SL^π	$\tilde{\beta}$	s^*	SL^π	$\tilde{\beta}$	s^*	SL^π	$\tilde{\beta}$	s^*	SL^π
2.5e3	(4,3,2,2,4,2,4)	78%	2.5e3	(4,3,2,2,3,2,6)	86%	2.5e3	(5,3,2,1,4,2,6)	91%	2.5e3	(4,2,1,3,4,2,8)	95%
4.4e3	(4,3,2,2,4,2,4)	78%	6.3e2	(4,3,2,2,3,2,6)	86%	6.3e2	(5,3,2,2,3,2,6)	91%	6.3e2	(5,4,2,2,2,3,6)	95%
5.8e3	(4,3,2,2,4,2,4)	78%	1.6e2	(5,4,3,3,1,2,4)	86%	1.6e2	(5,4,3,4,1,2,4)	91%	1.6e2	(5,3,3,4,1,2,6)	95%
6.8e3	(4,3,2,2,4,2,4)	78%	3.9e1	(6,5,4,3,0,2,2)	85%	3.9e1	(6,4,3,4,0,2,4)	91%	3.9e1	(7,4,4,5,0,2,2)	93%
7.6e3	(4,3,2,2,4,2,4)	78%	9.8e0	(9,5,5,3,0,0,0)	75%	9.8e0	(10,6,5,2,0,0,0)	78%	9.8e0	(10,6,4,2,0,2,0)	85%
8.7e3	(4,3,2,2,4,2,4)	78%	1.7e1	(8,5,4,3,0,2,0)	80%	1.7e1	(8,4,4,5,0,2,0)	85%	2.4e0	(19,4,1,0,0,0,0)	34%
9.0e3	(4,3,2,2,4,2,4)	78%	1.2e1	(9,5,5,3,0,0,0)	75%	1.2e1	(9,6,4,2,0,2,0)	83%	4.3e0	(14,5,3,2,0,0,0)	57%
9.2e3	(4,3,2,2,4,2,4)	78%	1.3e1	(8,5,4,3,0,2,0)	80%	1.0e1	(10,6,5,2,0,0,0)	78%	5.6e0	(13,5,4,2,0,0,0)	57%
9.4e3	(4,3,2,2,4,2,4)	78%	1.2e1	(9,5,5,3,0,0,0)	75%	1.1e1	(9,6,4,2,0,2,0)	83%	6.7e0	(11,6,5,2,0,0,0)	79%
9.6e3	(4,3,2,2,4,2,4)	78%	1.2e1	(8,5,4,3,0,2,0)	80%	-	-	-	7.4e0	(11,6,5,2,0,0,0)	79%
9.7e3	(4,3,2,2,4,2,4)	78%	-	-	-	-	-	-	8.0e0	(11,6,5,2,0,0,0)	79%
-	-	-	1.2e1	(8,5,4,3,0,2,0)	80%	1.1e1	(9,6,4,2,0,2,0)	83%	9.8e0	(10,6,4,2,0,2,0)	85%
	31.0			30.6			31.9			33.2	

Table 1: Output of the staffing algorithm (with 3 skills)

	agent group g											
m	01	02	03	04	05	06	07	08	09	10	11	12
1	x		x	x	x		x	x	x		x	x
2			x			x	x	x			x	x
3		x		x		x	x		x	x	x	x
4					x					x		x
5								x	x	x	x	x

Table 2: Skills and policy of the 5-skill example

	time t													
m	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	440	440	440	440	440	440	440	440	440	440	440	440	440	440
2	140	240	340	440	540	490	440	440	490	540	440	340	240	140
3	440	340	440	340	440	340	440	340	440	340	440	340	440	340
4	340	390	440	490	540	540	515	490	465	440	415	390	365	340
5	340	365	390	415	440	465	490	515	540	515	490	465	440	415

Table 3: Arrival rates per hour of example with 5 skills

The service rates are $\mu_{m,g} = 12$ for all m and g. We require the overall service level to be at least 80%.

The arrival rates per hour are presented in Table 3. We apply the algorithm from Section 2.1 of [PBK06]. The staffing costs are $(K^1(s_{t,1}), K^2(s_{t,2}), \ldots) = (0.908, 0.954, 0.908, 1, 1, 1, 1, 1, 1,\ 0.954, 1, 1)$. Note that we add the subscript t to denote the dependency on the time period. Table 4 presents the staffing vectors that resulted from the staffing algorithm. The time unit is one hour and the period of the simulation runs in both methods is 9,600 hours.

In order to evaluate the staffing algorithm more extensively, and to have more comparisons to Cezik & L'Ecuyer [CL06], we consider staffing costs similar to theirs: $\big(K^1(s_{t,1}), K^2(s_{t,2}), \ldots\big) = (1, 1, 1.1, 1.1, 1.1, 1.1, 1.2, 1.2, 1.2, 1.2, 1.3, 1.4)$. Table 5 presents the results of both algorithms. The content and structure of the table is similar to the previous tables. The performance measures are the objective value, the CPU time, and the number of simulations. We can read from the table that the accuracy and the computation times are both good compared to the cutting plane method.

References

[CL06] M.T. Cezik and P. L'Ecuyer, *Staffing multi-skill call centers via linear programming and simulation*, To appear in Management Science, 2006.

	time t													
g	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Lagrange relaxation													
$-f$	143.0	148.8	171.1	176.2	198.6	188.4	192.7	184.6	197.3	188.9	185.1	165.0	160.7	140.5
CPU	11	9	12	15	9	13	13	14	11	15	15	12	14	11
#sim	41	33	35	45	23	34	35	40	29	41	40	36	44	39
	Cutting planes													
$-f$	145.7	150.2	173.2	177.8	201.0	190.6	196.2	187.8	198.9	190.3	186.8	167.1	161.7	141.8
CPU	130	189	141	205	206	147	131	257	155	167	278	162	105	95
#sim	89	114	78	142	108	76	63	103	76	89	143	130	87	88

Table 4: Staffing vectors of example with 5 skills

g	time t													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Lagrange relaxation													
$-f$	165.5	175.6	200.1	207.6	233.6	223.3	226.7	217.5	231.8	223.4	217.3	193.5	188.1	166.3
CPU	13	14	12	13	16	19	15	17	13	15	17	12	15	14
#sim	50	50	37	37	40	50	40	49	36	41	46	38	45	50
	Cutting planes													
$-f$	168.9	177.3	203.3	210.1	236.7	225.3	229.8	220.4	234.3	226.6	219.9	198.2	190.3	167.7
CPU	212	234	414	184	414	420	329	300	461	285	426	333	336	220
#sim	210	185	214	121	242	255	203	158	239	201	263	222	174	148

Table 5: Staffing vectors of example with 5 skills

[PBK06] S.A. Pot, S. Bhulai, and G.M. Koole, *A simple staffing method for multi-skill call centers*, Submitted, 2006.